

All computations are executed to FOUR decimal places

Question (1) [20 marks]

Given the following four-point data

x	0.25	0.5	1.25	1.5
$f(x)$	-0.5	1.0	1.5	3.0

- Estimate $f(0.45)$ by using a second degree Newton's interpolating polynomial
- Determine the two constants a and b such that the curve $f(x) = ax + b \sin(2\pi x)$ fits the above data in the least-square sense.
- Assume that $f''(x)$ can be approximated using the backward formula ($h > 0$)

$$f''(x) = \frac{1}{h^2} [c_1 f(x) + c_2 f(x-3h) + c_3 f(x-4h)] + \mathcal{O}(h^n)$$

- Determine the three constants c_1, c_2, c_3 and the number n
- Use this backward relation to find $f''(1.25)$, (you should use a suitable value for h)
- Is it possible to find $f''(0.5)$ and $f''(1.5)$ by using this formula? Explain.

Question (2) [30 marks]

- Given the equation

$$f(x) = x^2 - \sin(x) - 7 = 0$$

- Prove that there are two roots for this equation, one is negative and the other is positive.
- Find the positive root correct to three decimal places using Simple iteration method
- Find the negative root correct to three decimal places using Newton Raphson method

- Given the system

$$x + 4y + z = 1, \quad 2x + y + z = 5, \quad x + y + 4z = 4,$$

with the initial approximation $(x_0, y_0, z_0) = (2.0, 0, 0)$. Find the third approximation (x_3, y_3, z_3) using the two methods:

- Jacobi iteration method
 - Gauss-Seidel method
- Given the initial value problem $yy' + \sin(y) = x^2$, $y(0) = 1.0$. Find the approximate value of $y(0.2)$ by using
 - Taylor method of order two and $h = 0.1$
 - Runge-Kutta four method with $h = 0.2$